Creativity and Complexity
(CDT-12)

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Abstract

Creativity has been the subject of great interest and even fascination to humans. Though it can be often recognized, it is difficult to be defined in a quantitative manner. At the same time, it seems reasonable to associate creativity to innovation, benefit, and efficiency while avoiding complexity. In this work, we report a development that, starting from a recent approach to complexity as being proportional to the modeling and error implied costs, attempts to integrate these concepts into a possible quantification of creativity. Having obtained a provional expression of creativity in terms of innovation, benefit and complexity, we discuss how creative solution of problems can be understood as a multiple-objective optimization procedure in the solution space underlain by landscapes ofthese 3 properties. Incremental solution as well as an approach based on analogies are discussed A discussion about the implications of the proposed framework is also developed, and some interesting results are obtained, including the identification of the typically varying complexity landscape as one of the major barriers to creativity.

'Per una volta che hai assaporato il volo, camminerai sulla terra con gli occhi rivolti verso il cielo, perché là sei stato e là desidererai ardentemente ritornare.'

Leonardo da Vinci.

1 Introduction

To some extent, humans can be understood as modelers of their world, themselves, as well as the relationship between the world and themselves (e.g. [1]). Indeed, modeling abilities contribute to enhanced chances of survival, as well as integration and positive collaborations one another and with the environment.

The intrinsic importance of modeling for humans along milenia ultimately gave rise to science and the scientific method, which are formalized means of obtaining more complete and effective models. The effectiveness of these approaches led to the development of many scientific and technological areas, which change the worlds and ourselves in the process, such as the relatively recent information technology implications on our ways of life and communication.

Yet, despite all these successes, there are unavoidable limitations to modeling. These stem from the fact that models of the real world can never be complete because they cannot consider all possible interactions between the modeled system and the remainder of the universe. In a sense, models are never complete being, therefore, subjected to possible changes, extensions and revisions (e.g. [1, 2]).

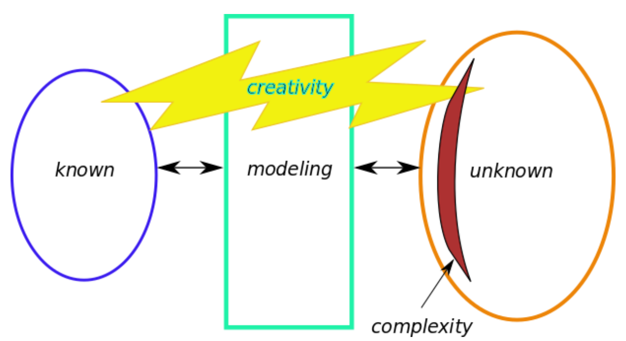

Figure 1: A possible framework relating the sets of known and unknown facts with modeling, complexity and creativity. The latter acts as a kind of spark providing a bridge, through complexity, into the realm of new knowledge.

In other words, the resources available for modeling natural phenomena – including scientific theories and methods, computational power, and our own cognitive abilities, are limited and less powerful than reality itself. In a sense, the scientific-technological advances are

typically achieved as a consequence of improvements in the underlying resources. A ubiquitous example consist in the development of electronic computing, through the invention of the transistor and integrated circuits, and the respective revolution in processing speed and memory capabilities that ultimately allowed personal computing and the internet.

Yet, at any given moment, an impending tension hovers between what is known and what remains to be known, a situation that goes back, at the least, to Socrates's saying that the more we know, the more we know that we do not know. This situation arises as a consequence of limitations on the modeling resources that would be otherwise necessary to pave the way to the next breakthroughs. In other words, *complexity is directly related to what we do not know yet*.

Recently [2], it has been argued that the complexity of a phenomenon or structure can be quantified in terms of the cost it takes to develop a respective model plus the cost of inferred prediction errors. As such, this approach not only acknowledges the fact that complexity is relative concept in space and time, but also that complexity is directly related to *modeling*.

Conquering knowledge from complexity can be achieved in many ways, including: (i) technological advances leading to new experiments, (ii) conceptual innovation providing new methods, (iii) perseverating hard working, as well as (iv) serendipity such as being in the right place in the right time (but it has also been said that luck favors the prepared mind).

In the present work, we argue that one of the main expedients potentially contributing to major knowledge advancements is *creativity*, our best allied while taming complexity. At a more fundamental level, we also posit that *complexity* and *creativity* are closely interrelated in an opposing way.

Figure 1 illustrates the considered framework interrelating modeling, creativity and complexity. We have two main sets: the known (left) and unknown (right) facts, which are intermediated by our modeling activities. Complexity acts as a kind of barrier or shield, making access to new knowledge difficult. Creativity then acts as a kind of spark, providing a brief portal between known and unknown facts, therefore enhancing our understanding of the universe.

This article is organized as follows. We start by briefly reviewing the recent cost-based approach to complexity [2], followed by a discussion of how efficiency can be expressed in terms of complexity. Then, we integrate all these elements into a provisional quantification of complexity, which is subsequently used for discussing the dynamics of creativity.

2 A Cost Approach to Complexity

In a recent work [2], we developed an approach to complexity of a structure or phenomenon that is based on the cost of respective modeling as well as on the cost implied by prediction errors. By *cost* it can understand not only monetary expenses, but also other resources such as time, equipment, personal, etc. involved in the respective research.

The challenging nature of complexity has motivated several ways to approach this critical problem. For instance, one can use the entropy of a given set of data in order to gauge its potential complexity. Yet, it is not uncommon to have high entropy data that is characterized by simplicity, such as data obtained from the uniform probability density, which has maximum entropy but is intrinsically simple. As an alternative, it is possible to consider the minimun coding length to infer the respective complexity, but then we have that distinct programming paradigms (e.g. recursive or not), not to mention different computer architectures, typically imply varying code lengths.

Many other interesting approaches that can be used to quantify complexity have been reported (e.g.see a brief review in [2]), and they are certainly very valuable in providing distinct perspectives about the complexity of a problem, phenomenon or structure. However, there are at least two issues that need to be accomodated into a wider complexity theory, and these concern the *relative* nature of complexity and the intrinsic human ability of *modeling* as the main approach to achieving scientific and technological advancements.

Regarding the relative nature of complexity, we need to take into account the fact that a same problem can be perceived as having different levels of complexity by two different people, and that can also vary along time. Not only different people have varying familiarity with specific subjects, but also the resources that are available in each situation are not fixed. For instance, performing some demanding calculation can be much more feasible, and therefore 'simple', for somebody with access to powerful computational resources than to someone on a more limited budget. Similarly, solving a hyperbolic equation will very likely look less complex after concluding an advanced calculus course. All in all, complexity is relative to *each human being* or modeling approach.

As for the modeling aspect of complexity, it follows immediately from the fact that human beings are intrinsically modelers of their environment and of themselves. We construct representations/models of world entities and phenomena all the time and everywhere, to the

point that language and science we ultimately developed as models of structures and actions in our world.

In modeling, complexity arises mainly as a consequence that every entity in the real world is infinitely related to other entities and effects in the universe. Just to provide a simple example, tides are a consequence of gravitational effects of our orbiting moon, being also influenced (though at a much smaller intensity) by other planets and stars. In brief, our universe is an intricate web of interrelationships and effects. Since models are necessarily limited in size and detail, much information needs to be left out in every developed model. Usually, it is precisely these missing pieces of information that can ultimately undermine a given model.

Perhaps the most direct relationship between modeling and complexity is the respective cost of the former. Indeed, more resources such as time and equipment can greatly contribute, typically, to achieving more complete and accurate models. Similarly, models developed with more limited resources tend to be less complete and, therefore, less accurate. The accuracy of model is, consequently, intrinsically related to the cost of modeling.

Modeling complexity is also related to the errors implied by predictions and operation of the model. However, instead of incorporating the magnitude of the prediction errors as part of the quantification of complexity, it is potentailly even more interesting to consider the cost of the respectively implied errors. Indeed, it is not difficult to imagine situations in which a relatively small error can lead to major undesirable consequences.

Thus, it has been suggested [2] that the complexity of a structure or phenomenon can be quantified in terms of some function of the cost of developing the respective model plus the cost of the implied prediction and operation errors. This viewpoint provides the starting point for our discussion of the relationship between complexity and creativity.

3 Efficiency

Creativity has been for a long time the subject of human marvel and concern. It was certainly experienced every time our ancestors discovered the usefulness of rocks or bones as tools or musica instruments. Creativity is also evident from the most ancient wall paintings in the deepest caves. There seems to be very little doubt that creativity is very dear to us, humans, as it not only surprises and delights us, but also can pave the way to major improvements in our standards of living.

Creativity while solving a problem is potentially related to the following two groups or properties: (a) innovation/originality / surprise; and (b) efficiency / optimization / high performance.

It is interesting to observe that these two groups or properties, henceforth identified respectively as *innovation* and *efficiency* are neither necessarily exclusive nor correlated. Indeed, we can efficiency without innovation (e.g. the wheel) or innovation devoid of efficiency (think of an example). Here, we understand that creativity is characterized by the *co-existence* or *positive correlation* between these two properties: *innovation* and *efficiency* when solving some problem.

Several examples of creativity can be recalled or found in the literature, so here we give just one example corresponding to the already mentioned discovery, by our long gone ancestors, that flutes can be made out of bones.

Let's now have a closer look at these two key elements: innovation and efficiency.

Innovation speaks for itself as being something new, original, on the face of the state of already existing knowledge or solutions. Observe that innovation is always relative to a specific set of already existing reference objects or concepts. Mathematically, it is interesting to quantify innovation in terms of the average *distance* between the new solution and each of the solutions already found in the reference set (see also [3]).

In order to try to define a metrics of innovation, it is first necessary to set up a respective metric space. This can be conceptualized as an $N-$dimensional space Ω so that each of its points corresponds to the *state* of a possible solution. In other words, every possible situation has its features mapped on each of the axes of Ω.

For instance, in case we seeking the best cover to a specific box, we could represent putative covers in terms of their length and width, giving rise to a respective $2-$dimensional space Ω. More general problems can involve many more dimensions. However, for simplicity's sake, we henceforth limit our attention to $2-$dimensional solution spaces, indicating each of the 2 axes in terms of the respective variables x and y. It is also necessary that the so-defined *solution space* Ω be a metric space, as in the case in this example, so that we can define distances between its points.

The use of distance to quantify innovation can now be illustrated in Figure 2, where the already existing solutions are represented as blue squares, and 4 new solutions are shown (green, cyan, orange and red). The first column in the legend gives the average Euclidean distances from each of these 4 new solutions to the blue points in reference set. I can be readily observed that this measurement provides a consistent quantification of the 'innovation' of each of these 4 solutions.

It should be borne in mind that many other measurements would be possible, such as using alternative types

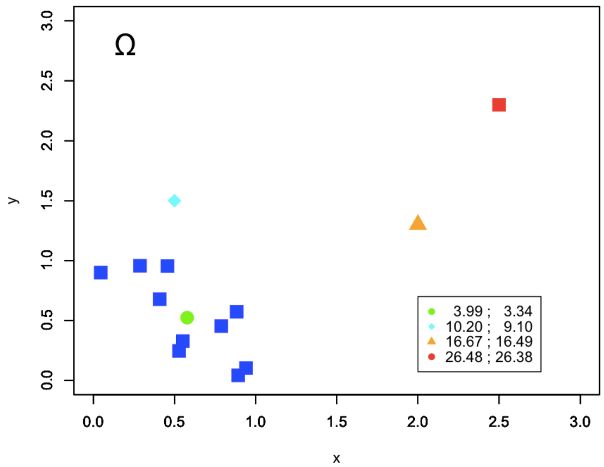

Figure 2: Solutions of a specific problem can be represented as points in a metric space $\Omega = [x, y]$. The innovation of new solutions with respect to an existing set of solutions (represented in blue) can be gauged in terms of the average distance between the point representing each new solution and the points in the reference set, as represented in the first column of the legend. It is also possible to use linear superimposition of potential fields emanating from the reference points onto each new solution (second column in the legend). See text for more information.

of distances (e.g. Mahalanobis), or field interactions. For instance, it is possible to define a scalar field emanating from each of the solutions in the reference set, so that the innovation of each new solution can be quantified by the superimposition of these potential fields. Field interactions are particularly interesting because, unlike distance-based measurements, they can be obtained by linear combination of effects and are, therefore typically easier to be calculated and manipulated mathematically.

The second column in Figure 2 presents a measurements derived from the total electric potential V arising at each of the 4 new solutions as a consequence of superimposition of potential fields (analogous to the electric potential) generated by each of the solutions in the reference set. As such fields tend to decrease with the distance, we used the reciprocal of the potential (i.e. $1/V$), multiplied by 100 for the sake of allowing a more direct comparison. In this specific example, the obtained results exhibited a very good agreement with the previously obtained average Euclidean distances.

Having briefly discussed how innovation can be quantified, let's turn our attention to the concept of *efficiency*, which is potentially more intricate and relative to some *merit figure*, reflecting the benefits implied by the solution. For instance, we can be interested in a computer that is faster, demands less power consumption, or is more robust. Sometimes, the efficiency component of a creative solution can also refer to some *aesthetic* property, such as balance, usability, or to its ability to cause surprise, not to mention design or even entertainment considerations.

All in all, the efficiency of a given solution to a problem can be quantified with respect to some specific merit figure of interest, which we will henceforth call *beneft* for simplicity's sake. However, this is not enough to define efficiency, as it does not take into account the expenses that need to be invested while obtaining the solution. These expenses, or costs for short, may have to do with the development time, or the number of people and equipment involved, environmental implications, etc. An efficient solution needs to be characterized by relatively high benefit and low cost. Mathematically, we can assume, as it has been often considered, that the efficiency is proportional to the ratio between modeling benefit and cost, i.e.

$$\text{efficiency} \propto \frac{\text{benefit}}{\text{modeling cost}} \tag{1}$$

Alternatively, in some situations in which attention needs to be placed on the magnitudes of operating and prediction errors, we can alternatively express efficiency as

$$\text{efficiency} \propto \frac{1}{(\text{error magnitude})\,(\text{modeling cost})} \tag{2}$$

Other similar expressions can be considered, such as taking the power of the terms in the above equation, or considering some other functions of these terms.

Interestingly, the here adopted quantification of complexity [2] can be derived from the previous efficiency expression by replacing the error magnitude by the *cost of error*, yielding

$$\textit{complexity} \propto (\text{error cost})\,(\text{modeling cost}) \tag{3}$$

So, complexity can be potentially understood as being proportional to the inverse of the efficiency in developing the model/solution. In other words, a complex problem does not tipically allow a solution that is both low cost and highly accurate. Actually, since the cost implied by operation/prediction errors is not necessarily related to the obtained benefit, we can complement our previous definition of efficiency by incorporating *both* these elements, yielding

$$\boxed{\text{efficiency} \propto \frac{\text{benefit}}{\text{complexity}} = \frac{\text{benefit}}{(\text{error cost})\,(\text{modeling cost})}} \tag{4}$$

This is a potentially more complete quantification of the efficiency of the solution of a problem, as it considers the cost of modeling, the cost implied by operation/prediction errors, and the expected benefits.

4 Creativity

Having discussed how *efficiency*, *benefit*, *cost* and *complexity* relate one another, we are now in a better position to approach and even try to define *creativity*.

Creativity has been the subject of interest for a very long time (e.g. [4]). More recent approaches include Guildford's distinction between *convergent* and *divergent* thinking, with the multiple paths contemplated by the latter potentially providing more grounds for creativity (e.g. [5]). Finke et al. proposed that creativity involves two phases: (i) construction of *pre-inventive* structures; and (ii) an *exploratory* phase (e.g. [6]). Another interesting approach has been described by A. Koestller, involving the concept of *bissociation* as referring to the intersection of two different frames of reference, allowing analogies, or *conceptual blending*, between different approaches (e.g. [7]). A quantitative insightful approach to creativity has been developed by R. Noller, including an expression of creativity as a function of *knowledge*, *imagination* and *evaluation* (e.g. [8]). The importance of associations between overlapping neuronal structures as a subsidy for the establishment of creative analogies has also been addressed (e.g. [9]).

An important point to be kept in mind is that, being a human-related concept, it is not possible to develop a fully precise or definitive approach to it. For instance, what is understood as creative by a person may not seem so much for another. Thus, the developments described in the following, mixing intuition and quantification, should be understood as being provisional and approximate.

There is little doubt that creativity in the solution of a given problem is directly proportional to the respectively *benefit* achieved, otherwise it would only be related to *innovation*, to which it is reasonable to understand it is also directly proportional.

As for the role of the complexity of a given solution, there are situations in which the development and/or implementation of a creative solution (e.g. the electric lamp) requires substantial costs, which suggest that creativity should not be related to cost. Indeed, often the implementation cost of some specific innovative solution is necessarily high, in the sense that no other less expensive but comparably efficient solution could be obtained.

Yet, even if the creative solution of a given problem unavoidably implies high costs, it is still interesting to take into account cost as a relative element allowing the identification of the less expensive creative solution.

It follows from the above reasoning about the relationship between creativity, benefit, innovation and complexity, that a possible respective working definition would be

$$\boxed{\textit{creativity} \propto \frac{\text{(benefit) (innovation)}}{\text{(complexity)}}} \tag{5}$$

which, by considering Equation 4, can be rewritten as

$$\boxed{\textit{creativity} \propto \text{(efficiency) (innovation)}} \tag{6}$$

Or, expanding complexity in terms of the the respectively involved costs (Eq. 3):

$$\boxed{\textit{creativity} \propto \frac{\text{(benefit) (innovation)}}{\text{(error cost) (modeling cost)}}} \tag{7}$$

This means that a creative solution would typically allow great benefit respectively to the problem specification, have low complexity (and therefore imply low development, implementation and operation costs), and be characterized by a good level of innovation.

This hypothesis is considered further in the following in terms of landscapes defined by innovation, benefit, complexity and creativity. It is interesting to observe that even if Equation 6 could be eventually verified to provide a good quantification of creativity, it would not directly tell us about how creativity can be achieved. However, as discussed in the following, it can still provide interesting insights about the dynamics of creativity.

5 Creative Solutions

Having a putative definition of creativity allows us to do interesting things, especially submitting it to considerations and applications, as well as trying to develop a respective *model* that could not just allow us to better understand creativity dynamics, but perhaps also to achieve it more frequently. Some of these possibilities are developed in the present section.

Let us start by considering a simple example of how Equation 6 can help us to approach creativity in an integrated and quantitative manner. In order to do so, we will use landscapes of innovation, benefit, and complexity of possible solutions to a given problem. These landscapes are maps from the solution space $\Omega = [x, y]$ into scalar respective scalar values of innovation, benefit, and complexity, as illustrated in Figure 3. For simplicity's sake, we adopt 2−dimensional landscapes, but it should be kept in mind that, in practice, Ω can have much higher dimensionality.

The first row of Figure 1 shows the innovation, benefit, complexity and creativity landscapes assigned to each possible solution $[x, y]$ in the solution space Ω. It is interesting to notice that the complexity landscape can eventually be split into two related landscapes corresponding to the modeling and operation costs.

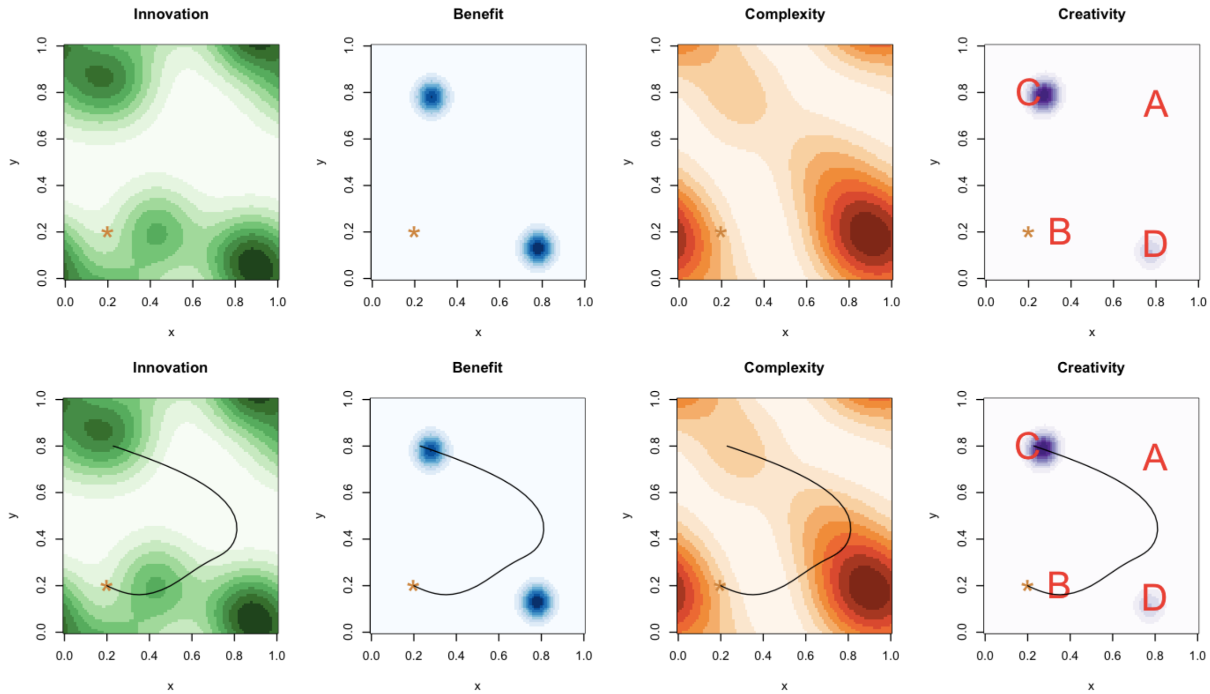

Figure 3: Hypothetical 2−dimensional landscapes of innovation, benefit, complexity, and creativity. The latter was obtained by applying Equation 6 on the three other landscapes. Observe that the domain of all these landscapes is the same, corresponding to the solution space $\Omega = [x, y]$. The first row of figures shows only the four landscapes, while the second row also incorporates a *trajetory* defined by a possible solution strategy. The asterisk identifies the assumed initial solution, and the trajectory unfolds along *all* the four landscapes. The intensity of the landscape values always increases from light to dark color tones.

It is important to observe that, in practice, we do not have access to the complete information in these four landscapes, but can only infer some of the values corresponding to already known solutions. As the efforts towards solution proceed, the landscapes are progressively unveiled.

Indeed, in case we knew the innovation, benefit and complexity landscapes, we could immediately apply Equation 6 and jump to the highest peak of creativity. However, for simplicity's sake, we will first consider that the four landscapes are known.

In this particular hypothetical example, we have an innovation landscape presenting several peaks and valleys, underlying solutions that are still unknown. Creativity is favored by high innovation, so we should aim at the respective peaks while taking into account the other two factors (i.e. benefit and complexity).

The benefit landscape in this particular example has two well defined, relatively narrow peaks with comparable intensities. They constitute possible solution targets. The complexity landscape is also characterized by several peaks and valleys, and the former are to be avoided during the unfolding of the solution.

Observe that the complexity landscape is, in this particular example, to depend on the initial solution (asterisk), but not on each followed trajectory. Therefore, the complexity values in this respective landscape for this first situation can be understood as corresponding to the *total* modeling/development cost.

The creativity landscape in this example was obtained by applying Equation 6 over the respective innovation, benefit and complexity maps. It reveals two peaks with quite different heights. Four regions are identified as A, B, C, and D in this landscape, corresponding to some particular situations often met in practice.

The situation A corresponds to a region with low innovation, low benefit and low complexity. Usually, a solution in this region is unlikely to be understood as being creative because it has little innovation and benefit, even though the complexity (and costs) is low.

The situation taking place at region B involves low innovation, low benefit and high complexity, being even less interesting as creative solutions.

Letter C identifies a region of the solution space Ω that is characterized by high innovation, high benefit and low complexity, being therefore the most creative region in this particular example. This is captured by the steep and intense peak obtained in the creativity landscape obtained by application of the adopted definition of creativity (i.e. Equation 6).

The region marked as D involves high innovation, high benefit, but at the expense of high complexity. As a con-

sequence, a relatively low respective peak was obtained in the creativity landscape.

Though simple, this example illustrates the ability of the adopted definition of creativity in identifying regions along the solution space that are particularly creative.

6 Incremental Approach

The previous example considered that we had access to all information in the four landscapes. Let's now consider the situation where our initial respective knowledge is limited, so that the unfolding of solutions involves performing explorations of the solution space while observing and considering the obtained innovation, benefit, complexity and respectively induced creativity.

Typically, one starts at a given preliminary solution, indicated by an asterisk in Figure 3. At this point, we know the respective innovation, benefit, and respective complexity. In the case of the example in our figure, we have that this initial solution is not particularly creative, so it is necessary to proceed searching for an improved solution. More formally, this typically corresponds to performing a *multiple-objective optimization*, more specifically a search along the solution space for a point which satisfies high innovation, high benefit and low complexity.

There are several ways to pursue such an optimization, going from developing incremental changes in the current solution – therefore defining *trajectories* in Ω – to jumping along this space according to some criterion or intuition. In the present work, we limit our attention to incremental trajectory-based creative solution pursuit. Such incremental approaches rely only on *local* information around the current tip of the trajectory.

Even under this constraint, there are still many possibilities to unfold our trajectory-based exploration of the solution space Ω. For instance, one can perform a 'greedy' approach in which we always move towards a new solution that increases innovation and benefit, while implying low complexity. This type of approach, however, cannot guarantee that we will reach the overall best solution in Ω, in the sense that it may converge to a *locally optimal* solution. For simplicity's sake, here we will consider this type of approach.

Going back to the example in Figure 3, having started from the solution marked by the asterisk, the illustrated trajectory departs in a direction leading to higher innovation and lower complexity. After some progress, the complexity starts increasing, and a change of direction is taken, proceeding now towards the upper-left portion of the solution space, moving away from complexity. Fortunately, this direction ends up into region C, characterized by high innovation, high benefit, and low complexity.

So far, we considered that the costs involved in the complexity landscape were total costs that did not depend on the taken trajectory, but only on the initial solution. This is seldom the case, as the costs tend to be accumulated along the trajectory. Consequently, the costs in the complexity landscape need to be integrated along the trajectory and, as a consequence, the costs can only increase monotonically. We will consider this situation further in the following section.

7 Dynamic Complexity

In the simplistic description of a possible solution to the problem in the previous example, we did not sconsider that the complexity is *accumulated* (more formally speaking, integrated) during the trajectory, especially in which refers the *total cost* of development. This is hardly the case in practice. Indeed, as one moves along the solution space, the complexity is not only accumulated, but it can also *change* along the problem solution.

For instance, it may happen that the knowledge acquired along the already pursued trajectory implies in a *reduction* of the overall complexity. It is also possible that as one moves along the solution space, the trajectory progress to a region in which further progress will only add complexity.

Figure 5 illustrates a more realistic scenario in which the complexity landscape changes as the trajectory unfolds.

This substantially more realistic situation, commonly found in practice, corresponds to one of the main difficulties in achieving creativity. The main problem is that, given that we have a current solution (x, y), as well as the knowledge about the innovation, benefit and complexity along the trajectory already taken, it is very difficult to predict how complexity will behave subsequently as the trajectory continues. In other words, the complexity landscape is not constant along the solution procedure, *but can vary substantially depending on the performed trajectory*, making it very difficult to choose a next solution step $(\Delta x, \Delta y)$.

For the sake of accuracy, it is important to observe that dynamic complexity landscapes are only allowed as a consequence of the fact that not all elements contributing to complexity have been allowed. In other words, the complexity is not only a function of x and y, but depends also on other variables not taken into consideration. As discussed in [1, 2], it is virtually impossible to completely model any real-world entity of phenomenon, including complexity. It is this incomplete mapping that allows the complexity landscape to change as we know more

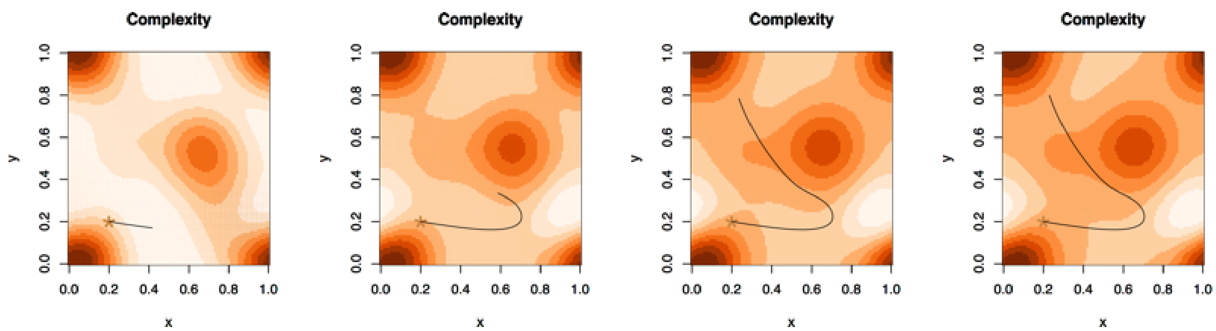

Figure 4: It is very common that the complexity landscape changes as the trajectory proceeds, as illustrated in this sequence.

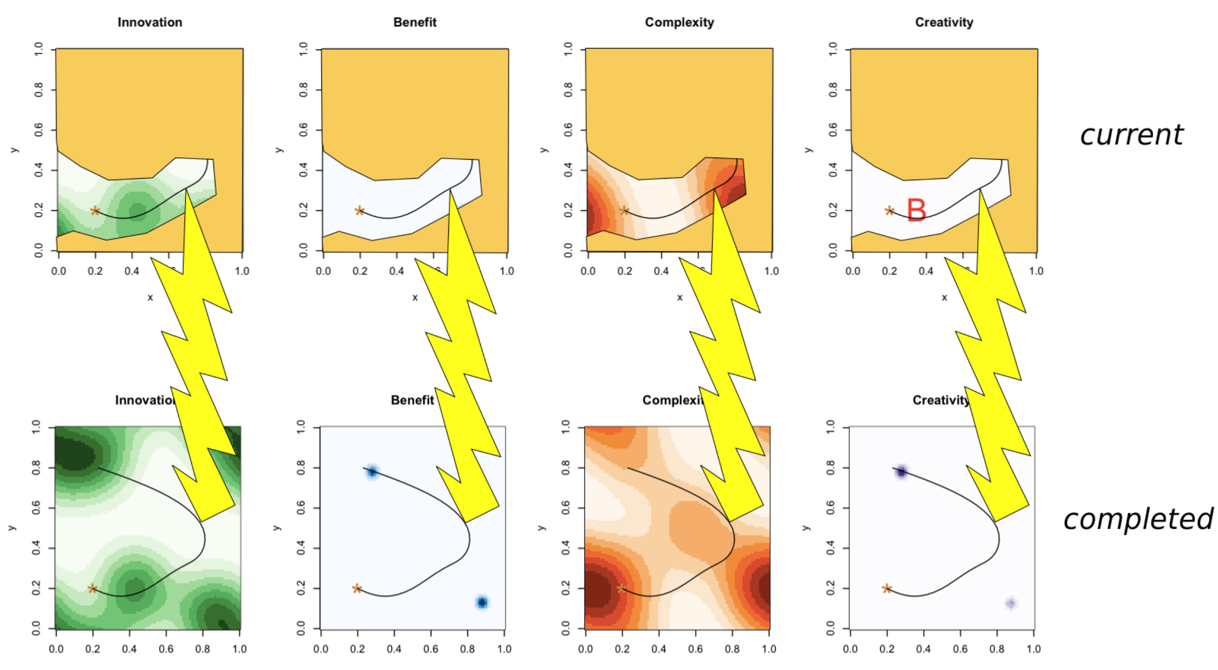

Figure 5: By establishing a correspondence or analogy between the current problem solution with a previous, completed solution, it is sometimes possible to make a jump into a more highly creative solution. The upper row illustrates a current state while solving a problem. Though limited, the knowledge about the innovation benefit, complexity and creativity landscapes can be compared to previous completed problems (lower row), allowing the identification of potentially valuable analogies.

about the problem, because these changes are caused by elements not considered in the mapping.

This phenomenon of varying complexity landscape probably accounts to why creativity is often such a challenge, being difficult to be achieved or planned. However, there is at least one approach that can be used in order to jump more directly into creativity, which is discussed next.

8 Shortcut Approach

Let's consider that we started from an initial solution (x_0, y_0) to a given specific problem and, therefore, have partial knowledge about the innovation, benefit and complexity landscapes (obtained along the already taken trajectory, and possibly considering also knowledge from other attempts).

Instead of incrementally probing the neighborhood of the current solution (x, y), it is possible to compare the currently unveiled landscapes with those obtained while solving other problems, related or not to the current one. This is illustrated in Figure 5 with respect to the current problem, and some other problem, already solved through a specific trajectory.

The upper row in this figure corresponds to the same landscapes as in Figure 3, while the lower row shows the complete landscapes obtained while solving another problem, possibly from a completely different area of knowledge.

The masked area in the landscapes along the first row of Figure 5 means that we do now know yet the information under those masks. However, we do have access to the landscapes values around the already performed trajectory. By comparing this information with other problems already solved (i.e. lower row), it is eventually possible to verify an similarity of the landscape features, therefore establishing an *analogy* between the two problems, which can in some cases pave the way to a *jump* more directly into a more creative region.

It is interesting to observe that the process of pursuing creativity through analogies can be greatly helped by adopting network science concepts and methods (e.g. [10, 11]). For instance, databases can be organized containing several problems with known creative solutions.

Each instance along the solution of a problem can be represented in terms of features extracted from the respectively available landscapes of innovation, benefit and complexity, and mapped into respective nodes. So, adjacent nodes belonging to different problem solutions offer promising venues for deriving analogies that can contribute in achieving more creative solutions.

9 Concluding Remarks

Creativity remains a very valuable asset for humans in our continuous quest for knowledge acquisition. In this text, we build up on a recent approach to complexity [2] in terms of development and error-implied costs in order to try to integrate these into a framework creativity.

Having briefly revised the cost-based approach to complexity [2], we discussed how efficiency can be extended from the more traditional formulation as benefit/cost so as to incorporate complexity, and we suggest that efficiency $\propto$ benefit/complexity.

Additional considerations led to the suggestion that creativity can be quantified as *creativity* $\propto \frac{\text{(benefit) (innovation)}}{\text{(complexity)}}$.

This formulation allowed a quantitative discussion to be developed while understanding the pursuit of a creative solution as an *multiple-objective optimization* problem. Though many more approaches could be taken into account, for simplicity's sake we constrained our attention to two specific alternatives: (a) developing trajectories along the solution space in an incremental, 'greedy' manner; and (b) through analogies between the already known landscapes and other previous, completed and successful, creative solutions.

We have seen that while the innovation and benefit landscapes tend to remain constant along the problem solution, the complexity landscape can vary in terms of the performed trajectory. We have argued that this happens as a consequence of the fact that the solution space is usually not enough to represent all the real-world effects influencing complexity, i.e. the complexity landscape is a non-bijective mapping from the real solution space.

These complexity variations can manifest themselves either as reductions caused by the fact that the knowledge already achieved facilitates the remaining path toward a good solution, or as augmentations in case we are heading toward a no-solution direction. It may also happen that the complexity landscape changes as a consequence of advances in other areas an problems, or as implied by budget reductions, etc.

This *dynamic* nature of the complexity landscape has been identified as one (possibly the main) of the significant barriers to quickly achieving creative solutions. A possible means to circumvent this problem is through analogies with other already successfully solved problems.

It is interesting to observe that the identification of the dynamic nature of the complexity landscape as a major limitation to creativity was only allowing in our study as a consequence of the explicit consideration and incorporation of complexity in the definition and discussion of

creativity and its possible dynamics.

All in all, we hope to have illustrated that the adopted quantitative approach to quantifying complexity, also incorporating complexity, can provide potentially valuable subsidies for better understanding as well as enhanced means for achieving creativity. The obtained framework, however, remains provisional and open to further modifications and complementations that may eventually be achieved, hopefully with the help of *creativity*.

Costa's Didactic Texts – CDTs

CDTs intend to be a halfway point between a formal scientific article and a dissemination text in the sense that they: (i) explain and illustrate concepts in a more informal, graphical and accessible way than the typical scientific article; and, at the same time, (ii) provide more in-depth mathematical developments than a more traditional dissemination work.

It is hoped that CDTs can also provide integration and new insights and analogies concerning the reported concepts and methods. We hope these characteristics will contribute to making CDTs interesting both to beginners as well as to more senior researchers.

Though CDTs are intended primarily for those who have some preliminary experience in the covered concepts, they can also be useful as summary of main topics and concepts to be learnt by other readers interested in the respective CDT theme.

Each CDT focuses on a few interrelated concepts. Though attempting to be relatively self-contained, CDTs also aim at being relatively short. Links to related material are provided in order to complement the covered subjects.

The complete set of CDTs can be found at: `https://www.researchgate.net/project/Costas-Didactic-Texts-CDTs`.

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References

[1] L. da F. Costa. Modeling: The human approach to science. Researchgate, 2019. `https://www.researchgate.net/publication/333389500_Modeling_The_Human_Approach_to_Science_CDT-8`. Online; accessed 03-June-2019.

[2] L. da F. Costa. Quantifying complexity. Researchgate, 2019. `https://www.researchgate.net/publication/332877069_Quantifying_Complexity_CDT-6`. Online; accessed 15-July-2019.

[3] V. Vieira, R. Fabbri, G. Travieso, O. N. Oliveira Jr, and L. da F. Costa. A quantitative approach to evolution of music and philosophy. *J. Stat. Mech.*, page P08010, 2012.

[4] Creativity. Wikipedia, 2019. `https://en.wikipedia.org/wiki/Creativity`. Online; accessed 15-July-2019.

[5] J. P. Guilford. *The nature of human intelligence.* McGraw Hill, 1967.

[6] S. M. Smith, T. B. Ward, R. A., and Finke. *The creative cognition approach.* MIT Press, 1995.

[7] A. Koestler. *The act of creation.* Hutchinson, 1964.

[8] R. B. Noller. Applied creativity: The creative studies project. *J. Creat. Behav.*, pages 276–294, 1972.

[9] L. Gabora and A. Saab. Creative interference and states of potentiality in analogy problem solving. In *Procs. of the Annual Meeting of the Cognitive Science Society*, Boston, MA, July 2011.

[10] L. da F. Costa. What is a complex network? Researchgate, 2018. `https://www.researchgate.net/publication/324312765_What_is_a_Complex_Network_CDT-2`. Online; accessed 19-May-2019.

[11] L. da F. Costa, F.A. Rodrigues, G. Travieso, and P.R. Villas Boas. Characterization of complex networks: A survey of measurements. *Advances in Physics*, (1):167–242, 2007.